Math 115

Winter 2017

Lecture 9

Class Quiz

① find slope of a line that contains
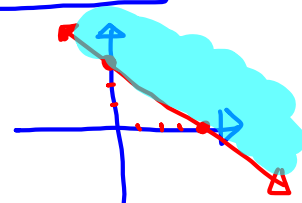
(-4, 1) and (0, 6) $m = \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{1 - 6}{-4 - 0}$

$= \dfrac{-5}{-4} = \boxed{\dfrac{5}{4}}$

② find eqn of a line that contains (4, -3) with

slope $\frac{1}{2}$. $y - y_1 = m(x - x_1)$ → $y + 3 = \frac{1}{2}x - 2$

$y - 3 = \frac{1}{2}(x - 4)$ $\boxed{y = \frac{1}{2}x - 5}$

③ Graph and shade: $y \geq \dfrac{-3}{4}x + 3$

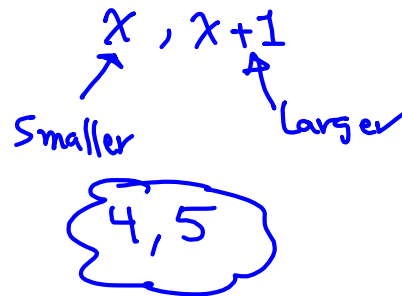① find __two cons. integers__ such that four times the larger one is equal to 5 times the smaller one.

$4 \cdot larger = 5 \cdot Smaller$

$4(x+1) = 5x$

$4x + 4 = 5x$

$4 = 5x - 4x$

$4 = x$

$x , x+1$

Smaller   Larger

4, 5

② Find two cons. odd integers such that 4 times the larger one is equal to the difference of 127 and 3 times the smaller one.

$4 \cdot larger = 127 - 3 \, Smaller$

$x \, \& \, x+2$

$4(x+2) = 127 - 3x$

$4x + 8 = 127 - 3x$

$4x + 3x = 127 - 8$

$7x = 119$

$x = \dfrac{119}{7}$

$x = 17$

17 & 19

find two cons. even integers such that when 8 times the smaller reduced by 3 times the larger, the result is equal to 124.

find the larger one.

$\rightarrow x$ & $x+2$

$8 \cdot smaller - 3 \cdot larger = 124$

$8 \cdot x - 3 \cdot (x+2) = 124$

$8x - 3x - 6 = 124$

$5x = 130$

$\rightarrow x = 26$

$26 + 2$

28

Solve

$\begin{cases} x + 3y = 7 \\ y = 2 - x \end{cases}$

$x + 3(2-x) = 7$

$x + 6 - 3x = 7$

$-2x = 1 \quad \boxed{x = \frac{-1}{2}}$

$y = 2 - \left(\frac{-1}{2}\right)$

$= 2 + \frac{1}{2} \quad \boxed{y = \frac{5}{2}}$

$\left(\frac{-1}{2}, \frac{5}{2}\right)$

System is consistent

Eqns are independent.

Solve

$$3 \begin{cases} 3x - 2y = -7 \\ 4x + 3y = 19 \end{cases} \Rightarrow \begin{cases} 9x - 6y = -21 \\ 8x + 6y = 38 \end{cases}$$

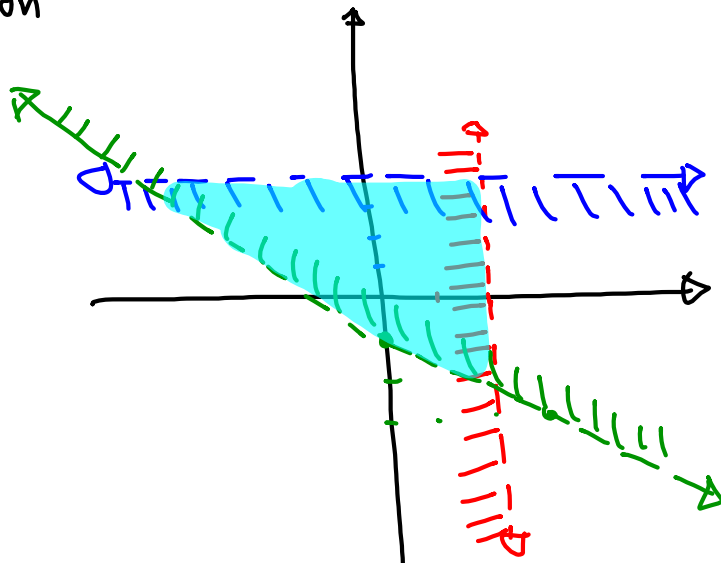$$17x = 17$$

$$\boxed{x = 1}$$

$$4(1) + 3y = 19$$

$$3y = 15 \rightarrow \boxed{y = 5} \qquad (1, 5)$$

Graph the Solution

$$\begin{cases} x < 2 \\ y < 4 \\ y > \frac{-2}{3}x - 1 \end{cases}$$

Ch.4

Exponential Rules

① x^{n} or exponent (Power

base

$$x^{n} = \underbrace{x \cdot x \cdot x \cdot x \cdots x}_{n \text{ times}}$$

$$5^{3} = 5 \cdot 5 \cdot 5$$

$$x^{7} = x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$$

$$(-3x)^{4} = (-3x) \cdot (-3x) \cdot (-3x)(-3x)$$

$$\left(\frac{5}{y^{2}}\right)^{2} = \frac{5}{y^{2}} \cdot \frac{5}{y^{2}}$$

② $x^{1} = x$

$$10^{1} = 10$$

$$(-7)^{1} = -7$$

$$(3x^{2})^{1} = 3x^{2}$$

$$(-5xy^{2})^{1} = -5xy^{2}$$

③ $x^{0} = 1$, $x \neq 0$

$$2^{0} = 1$$

$$(-10)^{0} = 1$$

$$\left(\frac{2}{3}\right)^{0} = 1$$

$$(4x^{2}y^{3})^{0} = 1$$

④ $x^m \cdot x^n = x^{m+n}$

$x^2 \cdot x^3 = x^{2+3} = x^5$

$(2x)^6 \cdot (2x)^4 = (2x)^{6+4} = (2x)^{10}$

$(-4y^3)^8 \cdot (-4y^3)^7 \cdot (-4y^3)^5 = (-4y^3)^{8+7+5} = (-4y^3)^{20}$

$\left(\dfrac{3x}{y^5}\right)^2 \cdot \left(\dfrac{3x}{y^5}\right)^{13} = \left(\dfrac{3x}{y^5}\right)^{15}$

⑤ $(x^m)^n = x^{m \cdot n}$

$(x^4)^2 = x^{4 \cdot 2} = x^8$

$(y^5)^3 \cdot y^6 = y^{15} \cdot y^6 = y^{15+6} = \boxed{y^{21}}$

$(x^6)^5 \cdot (x^8)^2 = x^{30} \cdot x^{16} = x^{30+16} = \boxed{x^{46}}$

⑥ $(xy)^n = x^n y^n$

$(2x)^4 = 2^4 \cdot x^4 = \boxed{16 x^4}$

$(-3x^2)^5 = (-3)^5 \cdot (x^2)^5 = \boxed{-243 x^{10}}$

$(4x^5 y^3)^3 = 4^3 (x^5)^3 (y^3)^3 = \boxed{64 x^{15} y^9}$

⑦ $\dfrac{x^m}{x^n} = x^{m-n}$

$\dfrac{x^7}{x^2} = x^{7-2} = \boxed{x^5}$

$\dfrac{x^{12} y^{18}}{x^4 y^{17}} = x^{12-4} y^{18-17} = x^8 y^1 = \boxed{x^8 y}$

$\dfrac{(x^6)^5}{x^{12} \cdot x^{18}} = \dfrac{x^{6 \cdot 5}}{x^{12+18}} = \dfrac{x^{30}}{x^{30}} = x^{30-30} = x^0 = \boxed{1}$

⑧ $\left(\dfrac{x}{y}\right)^n = \dfrac{x^n}{y^n}$

$\left(\dfrac{2}{3}\right)^4 = \dfrac{2^4}{3^4} = \boxed{\dfrac{16}{81}}$

$\left(\dfrac{-5}{x^2}\right)^3 = \dfrac{(-5)^3}{(x^2)^3} = \dfrac{-125}{x^{2\cdot3}}$

$= \boxed{\dfrac{-125}{x^6}}$

$\left(\dfrac{x^7 y^5}{z^6}\right)^4 = \dfrac{(x^7)^4 (y^5)^4}{(z^6)^4}$

$= \dfrac{x^{7\cdot4} \, y^{5\cdot4}}{z^{6\cdot4}}$

$= \boxed{\dfrac{x^{28} y^{20}}{z^{24}}}$

⑨ $x^{-n} = \dfrac{1}{x^n}$

$x^{-2} = \dfrac{1}{x^2}$

$(x^{-3})^4 = x^{-12} = \boxed{\dfrac{1}{x^{12}}}$

$(x^5)^{-6} = x^{-30} = \boxed{\dfrac{1}{x^{30}}}$

$(x^{-2})^{-8} \cdot (x^{10})^{-2}$

$= x^{(-2)(-8)} \cdot x^{10(-2)}$

$= x^{16} \cdot x^{-20}$

$= x^{16+(-20)} = x^{-4}$

$= \boxed{\dfrac{1}{x^4}}$

(10) $$\frac{x^{-n}}{y^{-m}} = \frac{y^m}{x^n}$$

$$\frac{x^{-2}}{y^{-5}} = \boxed{\frac{y^5}{x^2}}$$

$$\frac{x^7 y^{-4}}{x^{-3} y^6} = \frac{x^7 \, x^3}{y^6 \, y^4} = \boxed{\frac{x^{10}}{y^{10}}}$$

$$\frac{(x^5)^{-4} \cdot (y^{-3})^5}{x^6 \cdot y^{-25}} = \frac{x^{-20} \, y^{-15}}{x^6 \cdot y^{-25}} = \frac{y^{25}}{x^6 \, x^{20} \, y^{15}}$$

$$= \frac{y^{10}}{x^{26}}$$

(11) $$\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$$

$$\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \boxed{\frac{9}{4}}$$

$$\left(\frac{5}{x^4}\right)^{-3} = \left(\frac{x^4}{5}\right)^3 = \frac{x^{12}}{5^3} = \boxed{\frac{x^{12}}{125}}$$

$$\left(\frac{-2x^5}{y^6}\right)^{-4} = \left(\frac{y^6}{-2x^5}\right)^4 = \frac{(y^6)^4}{(-2)^4 (x^5)^4} = \boxed{\frac{y^{24}}{16x^{20}}}$$

Simplify:

$$\left(\frac{x^{-3} y^6}{z^{-8}}\right)^{-5} = \left(\frac{z^8 y^6}{x^3}\right)^{-5}$$

$$= \left(\frac{x^3}{z^8 y^6}\right)^5 = \frac{x^{15}}{z^{40} y^{30}}$$

$$(-8x^6)(5x^3)$$

$$= -40 \, x^9$$

$$\frac{-28 x^{12}}{4 x^{-3}} = -7x^{12-(-3)}$$

$$= -7x^{15}$$

Distribute & Simplify

1) $5(2x^2 - 3x + 6) - 3(3x^2 - 5x + 10)$

$= \underline{10x^2} - 15x + 30 \underline{-9x^2} + 15x - 30$

$= 1x^2 = \boxed{x^2}$

2) $-3x(5x + 7) + 5(3x^2 + 4x - 1) + x + 5$

$= -15x^2 - 21x + 15x^2 + 20x - 5 + x + 5$

$= \boxed{0}$

FOIL

→ Last ones

→ Inside ones

→ outside ones

→ first ones

$(x + 3)(x + 7) = x^2 + 7x + 3x + 21$

$= x^2 + 10x + 21$

$(2x - 3)(x + 5)$

$= 2x^2 + 10x - 3x - 15 = 2x^2 + 7x - 15$

Use FOIL to multiply:

① $(4x - 5)(3x - 7) = 12x^2 - 28x - 15x + 35$

$= 12x^2 - 43x + 35$

② $(2x + 9)(2x - 9)$

$= 4x^2 - 18x + 18x - 81$ $= 4x^2 - 81$

find an expression in simplest form for P & A

$P = 2L + 2W$

$A = LW$

5x-7

5x +7

$P = 2L + 2W$
$= 2(5x+7) + 2(5x-7)$
$= 10x + 14 + 10x - 14$
$= \boxed{20x}$
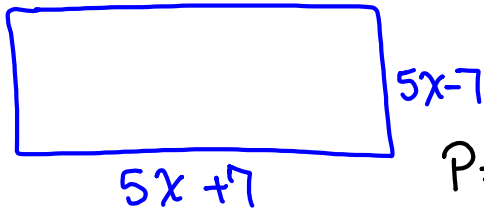
$A = LW$
$= (5x+7)(5x-7)$
$= 25x^2 - 35x + 35x - 49$
$= 25x^2 - 49$

Use foil to multiply

$(3x - 2)(9x^2 + 6x + 4)$

$= 27x^3 + 18x^2 + 12x - 18x^2 - 12x - 8$

$= \boxed{27x^3 - 8}$

Maria has $1.15 in nickels & Dimes only.

#of dimes is 1 more than 3 times # of nickels.

Use system of linear eqns to find how many of each.

N ⟶ # Nickels

D ⟶ # Dimes

$$\begin{cases} 5N + 10D = 115 \\ D = 3N + 1 \end{cases}$$

Use Subs. method

$5N + 10(3N+1) = 115$

$5N + 30N + 10 = 115$

$35N = 105$

$\boxed{N = 3}$

3 Nickels
&
10 Dimes

Jose has $230 in $10's & $20's only.

He has a total of 15 bills.

How many of each?

x ⟶ # $10 bills

y ⟶ # $20 bills

$$\begin{cases} x + y = 15 \\ \div 10 \quad 10x + 20y = 230 \end{cases}$$

$$-1 \begin{cases} x + y = 15 \\ x + 2y = 23 \end{cases}$$

$$y = 8$$

8 of $20 bills
&
7 of $10 bills

the number of kids was 3 more than 4 times the number of adults in a trip to local zoo.

kid's tkt → $3

Adult's tkt → $8

Total cost → $109

How many of each?

$$\begin{cases} K = 4A + 3 \\ 3K + 8A = 109 \end{cases}$$

Subs. method

$$3(4A + 3) + 8A = 109$$

$$12A + 9 + 8A = 109$$

$$20A = 100$$

$$\boxed{A = 5}$$

5 Adults
&
23 kids

Jack needs 100 pounds of candy @ $2.15 per pound.

He has unlimited supply of two types of candy, one @ $1.85/lb & the other one @ $2.25/lb. How many of each should he mix to obtain what he needs?

$$\begin{array}{c}\boxed{\$1.85} \\ x\end{array} + \begin{array}{c}\boxed{\$2.25} \\ y\end{array} = \begin{array}{c}\boxed{\$2.15} \\ 100\end{array}$$

$$\begin{cases} x + y = 100 \\ 1.85x + 2.25y = 2.15(100) \end{cases}$$

$$-37\begin{cases} x + y = 100 \\ 37x + 45y = 4300 \end{cases}$$

$$\begin{cases} -37x - 37y = -3700 \\ 37x + 45y = 4300 \end{cases}$$

$$8y = 600$$
$$\boxed{y = 75}$$

$$\begin{cases} x + y = 100 \\ 185x + 225y = 21500 \end{cases}$$

Divisible by 5

75 lb. of candy @ $2.25/lb. &
25 lb. of candy @ $1.85/lb.

Lisa works in a chem. lab.
She needs 20 liters of 18% alcohol soln.
She has unlimited supply of 12% & 20%
alcohol solutions.

Give ans in words.

| 12% | + | 20% | = | 18% |

x y 20

$$\begin{cases} x + y = 20 \\ \frac{12}{100}x + \frac{20}{100}y = \frac{18}{100} \cdot 20 \end{cases}$$

100

$$\begin{cases} x + y = 20 \\ 12x + 20y = 18(20) \end{cases}$$

÷4

$$\begin{cases} x + y = 20 \\ 3x + 5y = 90 \end{cases}$$

$$x = 5$$
$$y = 15$$

John need 100 liters of 82% acid Soln.
He has 40% acid Solution and a 100% pure acid
Solution.
How many liters of $\boxed{40\%}$ + $\boxed{100\%}$ = $\boxed{82\%}$
each? x y 100

$$\begin{cases} x + y = 100 \\ 100\begin{cases}\frac{40}{100}x + \frac{100}{100}y = \frac{82}{100}\cdot 100\end{cases}\end{cases} \Rightarrow \begin{cases} x + y = 100 \\ 40x + 100y = 8200 \end{cases}$$

Divisible by 20

$$-2\begin{cases} x + y = 100 \\ 2x + 5y = 410 \end{cases} \Rightarrow 3y = 210$$

$$\boxed{y = 70}$$

$\boxed{\text{70L of pure } \& \text{ 30L of 40\%}}$

Joe has 18L of 20% alcohol Solution.
How many liters of 40% alcohol solution
Should he mix to obtain a new Solution @
31% alcohol. Use one variable only.

Amount · Rate

$\boxed{20\%}$ + $\boxed{40\%}$ = $\boxed{31\%}$
   18   x $18+x$

$$\frac{20}{100}\cdot 18 + \frac{40}{100}\cdot x = \frac{31}{100}\cdot(18+x)$$

360 + 40x = 558 + 31x

40x − 31x = 558 − 360

9x = 198

$\boxed{x = 22}$

LCD = 100

$$360 + 40x = 31(18+x)$$

22 L

Due Thursday : whatever was due today

Work on WP 8, 9, and 10

Expect 2 Quizzes tomorrow.

Parallel & Perpendicular lines

System of linear eqns.